Ho-Lee model

The interese rate at time n is.

$R_n(w_1,\dots,w_n) = a_n + b_n \cdot \# H(w_1,\dots,w_n)$

The risk-neutral probability is $\tilde{p} = \frac{1}{2}$,

$a_0 = 0.05$, $a_1 = 0.045$, $a_2 = 0.04$, $b_1 = b_2 = 0.01$

$R_0 = 0.05$, $R_1(H) = 0.055$, $R_1(T) = 0.045$, $R_2(HH) = 0.06$, $R_2(HT) = 0.05$

$R_2(TH) = 0.05$, $R_2(TT) = 0.04$,

The discount process $D_1 = \frac{1}{1+R_0}$, $D_2 = \frac{1}{(1+R_0)(1+R_1)}$, $D_3 = \frac{1}{(1+R_0)(1+R_1)(1+R_2)}$.

$D_1(HH) = D_1(HT) = D_1(TH) = D_1(TT) = 0.9524$

$D_2(HH) = D_2(HT) = 0.9027$, $D_2(TH) = D_2(TT) = 0.9114$.

$D_3(HH) = 0.8516$, $D_3(HT) = 0.8597$, $D_3(TH) = 0.8680$, $D_3(TT) = 0.8763$.

then $B_{0,1} = \tilde{E} D_1 = 0.9524$, $B_{0,2} = \tilde{E} D_2 = 0.9071$, $B_{0,3} = \tilde{E} D_3 = 0.8639$

$B_{1,1} = 1$, $B_{1,2}(H) = \frac{1}{D_1(H)} \tilde{E}_1[D_2](H) = 0.9479$

$B_{1,2}(T) = \frac{1}{D_1(T)} = \tilde{E}_1[D_2](T) = 0.9569$

$B_{1,3}(H) = \frac{1}{D_1(H)} \tilde{E}_1[D_3](H) = 0.8985$

$B_{1,3}(T) = \frac{1}{D_1(T)} \tilde{E}_1[D_3](T) = 0.9158$

The time-two bond prices

$B_{2,2} = 1$, $B_{2,3}(HH) = \frac{1}{D_2(HH)} \tilde{E}_2[D_3](HH) = 0.9434$

$B_{2,3}(HT) = \frac{1}{D_2(HT)} \tilde{E}_2[D_3](HT) = 0.9524$

$B_{2,3}(TH) = \frac{1}{D_2(TH)} \tilde{E}_2[D_3](TH) = 0.9524$

$B_{2,3}(TT) = \frac{1}{D_2(TT)} \tilde{E}_2[D_3](TT) = 0.9615$.

For the 3-forward measure $\widetilde{P}^3$. The Radon-Nikodym derivative with respect to $\widetilde{P}$.

$$Z_3(HH) = \frac{D_3(HH)}{B_{0,3}} = 0.9858, \qquad Z_{3,3}(HT) = \frac{D_3(HT)}{B_{0,3}} = 0.9952$$

$$Z_{3,3}(TH) = \frac{D_3(TH)}{B_{0,3}} = 1.0047, \qquad Z_{3,3}(TT) = \frac{D_3(TT)}{B_{0,3}} = 1.0144$$

For $\widetilde{P}^3(w) = Z_{3,3}(w)\,\widetilde{P}(w)$,

$$\widetilde{P}^3(HHH) = Z_{3,3}(HH) \times \widetilde{P}(HHH) = 0.9858 \times \frac{1}{8} = 0.1232$$

$$\widetilde{P}^3(HHT) = Z_{3,3}(HH) \times \widetilde{P}(HHT) = 0.1232$$

$$\widetilde{P}^3(HTH) = \widetilde{P}^3(HTT) = 0.1244$$

$$\widetilde{P}^3(THH) = \widetilde{P}^3(THT) = 0.1256$$

$$\widetilde{P}^3(TTH) = \widetilde{P}^3(TTT) = 0.1268.$$